IICO - UASLP **PRIMERA EVALUACION** CALCULO 2

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Resuelva en el espacio dado lo que se indica justificando sus respuestas con un procedimiento matemático claro y válido. Sólo está permitido el uso de lápiz, borrador y sacapuntas (o puntillas).

1. (0.5 pts) Calcule la siguiente suma:

a) $S1 = \sum_{k=1}^{4}(k^3 - k^2 + 2k - 3) =$ $\left(\frac{n(n+1)}{2}\right)^2 - \frac{n(n+1)(2n+1)}{6} + 2\left(\frac{n(n+1)}{2}\right) - 3n = 100 - 30 + 20 - 12 = \underline{78}$

b) $S2 = \sum_{k=0}^{4}(-1/2)^{k-2} =$ $\left(-\frac{1}{2}\right)^{0-2} + \left(-\frac{1}{2}\right)^{1-2} + \left(-\frac{1}{2}\right)^{2-2} + \left(-\frac{1}{2}\right)^{3-2} + \left(-\frac{1}{2}\right)^{4-2} = \left(-\frac{1}{2}\right)^{-2} + \left(-\frac{1}{2}\right)^{-1} + \left(-\frac{1}{2}\right)^{0} + \left(-\frac{1}{2}\right)^{1} + \left(-\frac{1}{2}\right)^{2}$

$S_2 = (-2)^2 + (-2)^1 + 1 + \left(-\frac{1}{2}\right)^1 + \left(-\frac{1}{2}\right)^2 = 4 - 2 + 1 - \frac{1}{2} + \frac{1}{4} = 3 - \frac{1}{4} = \underline{\frac{11}{4}}$

2. (0.5 pts) Sea $\int_{-1}^{2}(x^2 + 3x - 1)\,dx$. Halle **a)** Números que satisfagan el **Teorema del Valor Medio** y **b)** el **Valor Medio** de $x^2 + 3x - 1$ en $[-1,2]$.

b-a) $f(z) = \int_{-1}^{2} f(x)\,dx$ Nota $f(x) = x^2 + 3x - 1$

$\int_{-1}^{2}(x^2+3x-1)dx = \left(\frac{x^3}{3} + \frac{3x^2}{2} - x\right)\Big|_{-1}^{2} = \left(\frac{8}{3} + 6 - 2\right) - \left(-\frac{1}{3} + \frac{3}{2} + 1\right) = 3 + 6 - 2 - \frac{3}{2} - 1 = 6 - \frac{3}{2} = \frac{9}{2}$

$((2) - (-1))f(z) = \frac{9}{2}$ $\therefore$ $3f(z) = \frac{9}{2}$ $\therefore$ $f(z) = \frac{9}{2(3)}$ $\therefore$ $\underline{f(z) = \frac{3}{2}}$ b)

Para a) $f(z) = z^2 + 3z - 1$ $\therefore$ $z^2 + 3z - 1 = \frac{3}{2}$ $\therefore$ $z^2 + 3z - \frac{5}{2} = 0$

$z = \frac{-3 \pm \sqrt{(3)^2 - 4(1)(-\frac{5}{2})}}{2(1)} = \frac{-3 \pm \sqrt{9 + 10}}{2} = \frac{-3 \pm \sqrt{19}}{2}$

Los numeros son: $z_1 = \frac{-3 + \sqrt{19}}{2}$

$z_2 = \frac{-3 - \sqrt{19}}{2}$ a)

3. (1 pts c/u) Evalúe la integral indicada:

$A = \int\left(2x^{-3} + \frac{3}{x^2} - 2x^2 - \sqrt[4]{x^3} + \frac{2}{\sqrt[5]{x^4}} + \frac{4}{x^{-3}}\right)dx$

$A = \int(2x^{-3} + 3x^{-2} - 2x^2 - x^{3/4} + 2x^{-4/5} + 4x^3)dx =$

$A = \frac{2x^{-2}}{-2} + \frac{3x^{-1}}{-1} - \frac{2x^3}{3} - \frac{x^{7/4}}{7/4} + \frac{2x^{1/5}}{1/5} + \frac{4x^4}{4} = \underline{-\frac{1}{x^2} - \frac{3}{x} - \frac{2}{3}x^3 - \frac{4}{7}x^{7/4} + 10x^{1/5} + x^4 + c}$

$B = \int(y + y^{-1})^2\,dy$

$B = \int(y^2 + 2(y)(y^{-1}) + y^{-2})\,dy = \int(y^2 + 2 + y^{-2})dy = \frac{y^3}{3} + 2y + \frac{y^{-1}}{-1} = \underline{\frac{1}{3}y^3 + 2y - \frac{1}{y} + c}$

$C = \int_{1}^{1} 3x^7\,(x^2 - x)^{2/3}\,dx$ $\underline{= 0}$

Justificación

$\int_a^a f(x)\,dx = F(x)\Big|_a^a = F(a) - F(a) = 0$

$$D = \int \frac{\sqrt[4]{1 - v^{-1}}}{v^2} dv = \int \frac{m^{1/4}}{v^2} v^2 dm = \frac{m^{5/4}}{\frac{5}{4}} = \frac{4}{5}(1 - v^{-1})^{5/4} + c$$

$m = 1 - v^{-1}$

$dm = + v^{-2} dv = \frac{1}{v^2} dv$

$$E = \int_1^9 \sqrt{2r + 7}\, dr = \int_{r=1}^{r=9} R^{1/2} \left(\tfrac{1}{2}\right) dR = \frac{1}{2} \frac{R^{3/2}}{\frac{3}{2}} \Big|_{r=1}^{r=9} = \frac{1}{3}(2r+7)^{3/2} \Big|_1^9$$

$R = 2r + 7$

$dR = 2\, dr$

$$E = \frac{1}{3}(2(9)+7)^{3/2} - \frac{1}{3}(2(1)+7)^{3/2} = \frac{1}{3}(25)^{3/2} - \frac{1}{3}(9)^{3/2} = \frac{125}{3} - \frac{27}{3} = \frac{98}{3}$$

$$E = 98/3$$

$$F = \int_0^1 \frac{z^2}{(1 + z^3)^2} dz = \int_{z=0}^{1} \frac{z^2}{U^2} \frac{1}{3z^2} dU = \frac{1}{3} \int_{z=0} U^{-2} dU = \frac{1}{3} \frac{U^{-1}}{-1} \Big|_{z=0}^{z=1} = -\frac{1}{3(1+z^3)} \Big|_0^1$$

$U = 1 + z^3$

$dU = 3z^2 dz$

$$F = \left[-\frac{1}{3(1+1^3)}\right] - \left[-\frac{1}{3(1+0^3)}\right] = -\frac{1}{6} + \frac{1}{3} = \frac{1}{6}$$

$$G = \int_1^2 \frac{s^2 + 2}{s^2} ds = \int_1^2 \left(\frac{s^2}{s^2} + \frac{2}{s^2}\right) ds = \int_1 (1 + 2s^{-2}) ds = \left(s + \frac{2s^{-1}}{-1}\right)\Big|_1^2$$

$$G = \left(s - \frac{2}{s}\right)\Big|_1^2 = \left(2 - \frac{2}{2}\right) - \left(1 - \frac{2}{1}\right) = 2 - 1 - 1 + 2 = 2$$

$$H = \frac{d}{dz} \int_3^{z^3} (x^2 + 1)^{10} dx = \frac{d}{dz}\left(F(x)\Big|_3^{z^3}\right) = \frac{d}{dz}\left(F(z^3) - F(3)\right) = f(z^3) \frac{d}{dz} z^3 - 0$$

$f(x) = (x^2+1)^{10}$ — Regla de la cadena — cte

$$H = ((z^3)^2 + 1)^{10} (3z^2) = 3z^2 (z^6 + 1)^{10}$$

$$I = \int_0^7 \frac{d}{dx}\left(\frac{x^2}{\sqrt{3x + 4}}\right) dx = \frac{x^2}{\sqrt{3x+4}} \Big|_0^7 = \frac{7^2}{\sqrt{3(7)+4}} - \frac{0^2}{\sqrt{3(0)+4}} = \frac{49}{5}$$

$$\int \frac{d}{dx}(\text{algo})\, dx = \text{algo} + c$$